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MODELING THE HEAT TRANSFER PROCESS IN AN INFILTRATED GRANULAR BED WITH REGARD FOR THE TEMPERATURE DIFFERENCE OF THE PHASES

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Based on the solutions of the "internal" and "external" problems of a gas flow in a granular bed, expressions are obtained for calculating the interphase heat transfer coefficient and the degree of gas flow nonuniformity.

As is known [1], the use of the one-temperature model of a disperse medium

$$(C_f\rho_f\varepsilon + C_s\rho_s(1-\varepsilon))\frac{\partial T}{\partial t} + (\pm C_f\rho_f u \pm C_s\rho_s u_s)\frac{\partial T}{\partial z} = \lambda_{ef}\frac{\partial^2 T}{\partial x_i^2} \quad (1)$$

may substantially distort the actual picture of heat transfer in many nonstationary processes associated with heat treatment of disperse materials. In such cases it is more effective to use two-temperature models that take into account the temperature difference of the phases and interphase heat transfer.

Under the assumption of isotropy of the thermal conductivities of the phases and thermally fine particles the energy equations may be written in the form

$$C_f\rho_f\varepsilon\frac{\partial T_f}{\partial t} \pm C_f\rho_f u\frac{\partial T_f}{\partial z} = \lambda_f\frac{\partial^2 T_f}{\partial x_i^2} + \frac{6A(1-\varepsilon)}{d}\alpha(T_s - T_f), \quad (2)$$

$$C_s\rho_s(1-\varepsilon)\frac{\partial T_s}{\partial t} \pm C_s\rho_s u_s\frac{\partial T_s}{\partial z} = \lambda_s\frac{\partial^2 T_s}{\partial x_i^2} - \frac{6A(1-\varepsilon)}{d}\alpha(T_s - T_f). \quad (3)$$

System (2), (3) contains four a priori unknown coefficients that determine the intensity of conductive and interphase heat transfer: λ_f, λ_s, A, and α. With this many parameters we must naturally seek reasonable simplifications to reduce this number to a minimum.

In [2, 3] the convective component of the known thermal conductivity of the granular bed λ_{ef} was used for λ_f, and its conductive component (independent of the relative velocity of the phases). A similar procedure was used in [4], where an infiltrated granular medium was considered to consist of the bed skeleton (particles and impassable zones near the points of contact of them) and continuous-flow zones. In [5, 6], λ_f was equated to λ_{ef}, and the contact thermal conductivity λ_{ct} for the skeleton of particles, determined from measurements of the thermal conductivity of the filling in a vacuum, was used for λ_s.

Extensive literature is devoted to the determination of interphase heat transfer coefficients (see the surveys in [7–9]). Although the scatter of experimental data is great, it is reliably established that heat transfer coefficients may be divided conventionally into two groups: at Re > 100, when the measured α values approximately follow known dependences [7, 9] for a single fixed particle; at Re < 100, when the α values are sometimes several orders of magnitude lower than for a single particle (see Fig. 1). The values of the first group are called "actual," and those of the second group are called "effective" [9]. So far, no sufficiently satisfactory quantitative explanation

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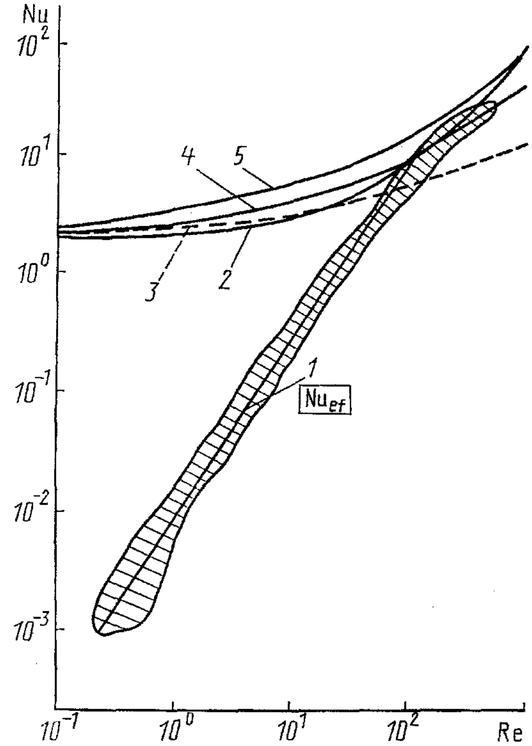

Fig. 1. Dependence Nu = f(Re) for an infiltrated granular bed: 1) $Nu_{ef} = 0.01Re^{1.57}$ (a generalization of the experimental data in the hatched region); 2) calculation by (7); 3) calculation for a single sphere by the equation $Nu = 2 + 0.6Re^{1/2}Pr^{1/3}$; 4) calculation by (7b); 5) calculation by (7a).

exists for the distinctive heat transfer "crisis" at Re $\approx$ 100 (qualitative arguments may be found in [8, 9]). It is noted that the substantial decrease in α in the range $R < 100$ is caused by two main factors, namely, the neglect of the effective thermal conductivity of the gas and nonuniform laminar gas flow past the particles, which leads to a decrease the active interphase surface. The "actual" and "effective" heat transfer coefficients are considered theoretically in [5, 10].

The coefficient A – the part of the total interphase surface that participates in heat transfer – has hardly been studied. In the literature [8] only qualitative estimates of this parameter are reported. Therefore in [2–6] A is simply assumed equal to 1.

The present work is devoted to determining the coefficients α and A entering Eqs. (2) and (3) and to investigating the effect of λ_{ef} and A on the effective interphase heat transfer coefficient in order to have grounds for using some particular value of it in describing nonstationary heat transfer processes in a system.

We shall analyze the regularities of interphase heat transfer on the basis of the "internal" problem (a gas flow in pore channels). We assume the diameter of the equivalent channel to be [1]: $D_{eq} = 2R = 2d\varepsilon/3(1 - \varepsilon)$; its height is evaluated from the crookedness of pores k: $H = kd$. We assume that $k = 1.5$ [1].*

The problem on interphase heat transfer is formulated mathematically as

$$\frac{\partial^2\theta}{\partial\eta^2} + \frac{1}{\eta}\frac{\partial\theta}{\partial\eta} + \left(\frac{R}{H}\right)^2\frac{\partial^2\theta}{\partial\xi^2} - \mathrm{Pe}\frac{\partial\theta}{\partial\xi} = 0,$$

$$\frac{\partial\theta(\xi, 0)}{\partial\eta} = 0, \quad \theta(\xi, 1) = 1, \tag{4}$$

* Such assumptions actually correspond to the absence of the stagnant zones that occur due to local nonuniformities of the gas flow, and they correspond to $A = 1$.

$$\frac{\partial\theta(0,\eta)}{\partial\xi}+\mathrm{Pe}\left(\frac{H}{R}\right)^2\theta(0,\eta)=0\,,\quad \frac{\partial\theta(1,\eta)}{\partial\xi}=0\,,$$
$$\theta=(T-T_0)/(T_w-T_0)\,. \tag{5}$$

The expression for the effective thermal conductivity of a gas in the interpore space is given in the standard form $\lambda_f^h=\lambda_f^k+BC_f\rho_f ud/\varepsilon$. The coefficient B is assumed equal to 0.0061 in accordance with the expression $\lambda_f^h=\lambda_f^k+0.0061C_f\rho_j ud/\varepsilon$ for the thermal conductivity of a gas film near a particle surface. This expression is a result of processing experimental data on external heat transfer within the framework of our two-zone model [11].*

System (4), (5) was solved numerically by a time-dependent method [12]. The nonstationary problem corresponding to this system was approximated by an implicit scheme realized by the method of matrix factorization [12], which was absolutely stable in the given case. The interphase heat transfer coefficient was calculated by the formula

$$\alpha=\frac{\lambda_f^h}{T_w-T_0}\left.\frac{\partial T}{\partial r}\right|_{r=R}. \tag{6}$$

Results of numerical calculations together with known experimental data on heat transfer in motionless granular beds reported in [7–9] are shown in Fig. 1. The numerical solution is approximated in the form

$$\mathrm{Nu}=2+0.042\,\mathrm{Re}/\varepsilon\,. \tag{7}$$

Formula (7) at high Re numbers, when the first summand may be neglected, agrees very well with the formulas of Timofeev [13] $\mathrm{Nu}=0.042\,\mathrm{Re}/\varepsilon$ and Gol'dshtik [5] $\mathrm{Ni}=0.043\,\mathrm{Re}/\varepsilon$.

Taking into consideration the absence of experimental data on α at low Re in the literature, it is expedient, in order to increase the reliability of the theoretical results, to have Nu values based on the "external" problem of a flow past the particles in a bed. An approximate solution, but one entirely suitable for our purposes, may be obtained using an expression obtained earlier [14] for the heat transfer coefficient of a vertical cylinder of diameter $D \gg d$ in an infiltrated bed. For this, it is sufficient to consider the asymptotics of the solution at $D\to d$ (the height of the cylinder was also assumed equal to d). In accordance with the recommendations given in [15], the thickness of the boundary gas film was assumed equal to $0.05d$. With regard for the remarks made, we derive the following relation for evaluation of the interphase heat transfer**:

$$\mathrm{Nu}=2k_0^0\,\frac{k^0\sqrt{\mathrm{Pe}_s^*}+0.1\,\mathrm{Pe}_f^*/K^*}{1+0.1k^0\sqrt{\mathrm{Pe}_s^*}\,K^*}\,K^*\,, \tag{7a}$$

where

$$\mathrm{Pe}_s^*=\frac{\mathrm{Re\,Pr}}{4\,k_0^0k^0}\,;\quad \mathrm{Pe}_f^*=\frac{\mathrm{Re\,Pr}}{2\varepsilon\,k_0^0}\,;\quad k_0^0=\frac{\lambda_f^h}{\lambda_f^k}\,;$$

$$k^0=\frac{\lambda_{ef}}{\lambda_f^h}\,;\quad K^*=K_1\left(1.1\sqrt{\mathrm{Pe}_s^*}\right)/K_0\left(1.1\sqrt{\mathrm{Pe}_s^*}\right).$$

* Such a choice of the value of λ_f^h in the interpore space will obviously result in somewhat underestimated values of α (see below).

** Note that formula (7a) is obtained for high Re. The possibility of using it for calculations at low Re is substantiated in [14, Fig. 4b].

Calculations by (7a) are also given in Fig. 1. The sought function Nu(Re) was obtained by the least-square method. In doing so, at low Re expressions (7), (7a) and known formulas for heat transfer of a single sphere were used as "experimental," and at Re > 100 the requirement of maximum similarity to the known dependences $\mathrm{Nu} = 2 + 1.8\,\mathrm{Re}^{1/2}\mathrm{Pr}^{1/3}$ [7] and $\mathrm{Nu} = 0.4(\mathrm{Re}/\varepsilon^{2/3}\mathrm{Pr}^{1/3}$ [9] was achieved.

The final form of the formula for the interphase heat transfer coefficient is

$$\mathrm{Nu} = 2 + 0.56\,\mathrm{Re}^{2/3}\,. \tag{7b}$$

Correlation (7b) is used below to determine the coefficient A entering (2) and (3).

We now consider, as a model, a stationary heat transfer process of a gas-infiltrated granular bed in terms of two-temperature system (12), (3). As in [5, 6], we assume that $\lambda_f = \lambda_{ef}$, $\lambda_s = \lambda_{ct}$. According to [16], λ_{ct} may be neglected at $\lambda_s^k/\lambda_f^k < 40$. In such an approximation, model (2), (3) acquires the form

$$C_f\rho_f u \frac{dT_f}{dz} = \lambda_{ef}\frac{d^2T_f}{dz^2} + \frac{6A\,(1-\varepsilon)}{d}\,\alpha\,(T_s - T_f)\,, \tag{8}$$

$$C_s\rho_s u_s \frac{dT_s}{dz} = \frac{6A\,(1-\varepsilon)}{d}\,\alpha\,(T_s - T_f)\,. \tag{9}$$

System (8), (9) describes a widely used process of gradientless heating (cooling) of a disperse material moving opposite to a gas. The single unknown parameter A, determining the active interphase surface, may be found by comparing the "theoretical" temperature distributions, obtained from solution of (8), (9), with the "experimental" profiles obtained from the equations

$$C_f\rho_f u \frac{\partial T_f}{\partial z} = \frac{6A\,(1-\varepsilon)}{d}\,\alpha_{ef}\,(T_s - T_f)\,, \tag{10}$$

$$C_s\rho_s u_s \frac{\partial T_s}{\partial z} = \frac{6A\,(1-\varepsilon)}{d}\,\alpha_{ef}\,(T_s - T_f)\,, \tag{11}$$

which, as is known [7–9], are used to determine the effective heat transfer coefficient from the experimental temperature profiles of a gas.

We generalized rather numerous experimental data [7–9] on α_{ef} entering (10) and (11) in the form

$$\mathrm{Nu}_{ef} = 0.01\,\mathrm{Re}^{1.57}\,, \quad \mathrm{Re} \le 100\,. \tag{12}$$

The values of α_{ef} calculated by (12) were used to obtain the "experimental" temperature profiles of the phases by (10), (11).

We now write systems (8), (9) and (10), (11) with the corresponding boundary conditions in dimensionless form:

$$\mathrm{Pe}_f\frac{d\theta_f}{d\xi} = \frac{d^2\theta_f}{d\xi^2} + A\,\mathrm{Pe}^*\,(\theta_s - \theta_f)\,, \tag{13}$$

$$\mathrm{Pe}_s\frac{d\theta_s}{d\xi} = A\,\mathrm{Pe}^*\,(\theta_s - \theta_f)\,, \tag{14}$$

$$\theta_s\,(1) = 1\,, \tag{15}$$

$$-d\theta_f/d\xi + \mathrm{Pe}_f\theta_f = 0\,, \quad \xi = 0; \quad d\theta_f/d\xi = 0\,, \quad \xi = 1 \quad \text{(the Dankwerts conditions)} \tag{16}$$

and

$$\frac{d\theta_f}{d\xi} = \beta_f(\theta_s - \theta_f)\,, \tag{17}$$

$$\frac{d\theta_s}{d\xi} = \beta_s(\theta_s - \theta_f)\,, \tag{18}$$

$$\theta_s(1) = 1\,, \tag{19}$$

$$\theta_f(0) = 0\,. \tag{20}$$

For systems of a gas and solid particles the solutions of (13)-(16) and (17)-(20) are as follows: for system (13)-(16)

$$\theta_f = 1 + B_f \exp(k_3\xi)\,, \quad \theta_s = 1 + B_s \exp(k_3\xi)\,, \tag{21}$$

where

$$k_3 = \left(\frac{A\,\mathrm{Pe}^*}{\mathrm{Pe}_s} + \mathrm{Pe}_f\right) \Big/ 2 - \sqrt{\left(\left(\frac{A\,\mathrm{Pe}^*}{\mathrm{Pe}_s} - \mathrm{Pe}_f\right)^2 \Big/ 4 + A\,\mathrm{Pe}^*\right)}\,,$$

$$B_s = \mathrm{Pe}_f/(\bar{k}_3k_3 - \mathrm{Pe}_f\bar{k}_3)\,; \quad B_f = \bar{k}_3B_s\,; \quad \bar{k}_3 = (A\,\mathrm{Pe}^* - k_3\,\mathrm{Pe}_s)/A\,\mathrm{Pe}^*\,;$$

for system (17)-(20)

$$\theta_f = 1 - \exp[(\beta_s - \beta_f)\,\xi]\,, \quad \theta_s = 1 - \beta_* \exp[(\beta_s - \beta_f)\,\xi]\,. \tag{22}$$

A comparison of "theoretical" (21) and "experimental" (22) functions was made by comparing the exponents.* The requirement of their equality allowed a simple formula to be obtained for determining the sought parameter A:

$$A = \frac{(\beta_s - \beta_f)^2 - (\beta_s - \beta_f)\,\mathrm{Pe}_f}{\mathrm{Pe}^*\left(1 - \dfrac{\mathrm{Pe}_f}{\mathrm{Pe}_s} + \dfrac{\beta_s - \beta_f}{\mathrm{Pe}_s}\right)}\,. \tag{23}$$

The values of A calculated by (23) were approximated by the following dependences:

$$A = 0.006\,\mathrm{Re}^{1.32}\,, \quad \mathrm{Re} \le 10\,; \quad A = 0.027\,\mathrm{Re}^{0.88}\,,$$
$$10 < \mathrm{Re} \le 100\,; \quad A = 1\,, \quad \mathrm{Re} > 100\,. \tag{24}$$

In order of magnitude, the values of A are fairly consistent with estimates obtained by S. S. Zabrodskii [8] for a fluidized bed using the microbreak-out model of excess gas. At Re > 100(the region of a sufficiently developed turbulent regime) the nonuniformity of gas flow distribution no longer exists. Under these conditions, as

* Because of a fundamental difference in boundary conditions (16) and (20) it is impossible to demand the coincidence of the functions θ_f themselves that are calculated by (21) and (22). Their difference is particularly great at $\xi = 0$ for low Re (low Pe_f).

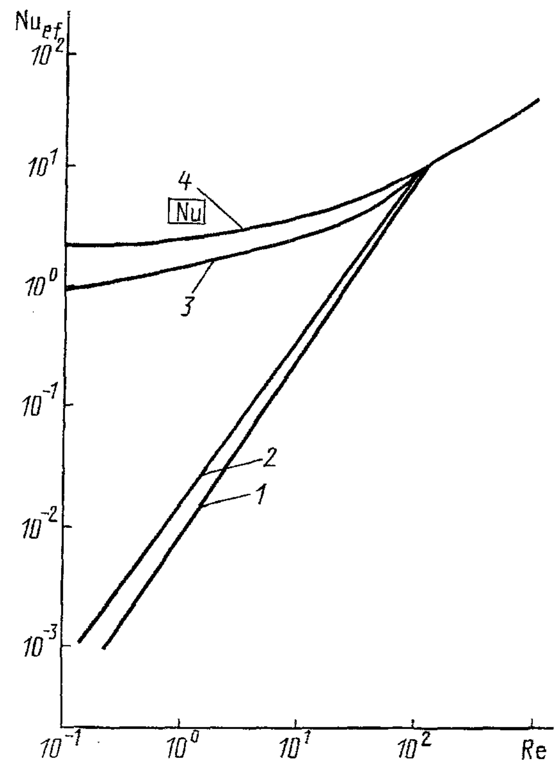

Fig. 2. Effective interphase heat transfer coefficients: 1) $Nu_{ef} = 0.01Re^{1.57}$ (generalization of the experimental data); 2) Nu''_{ef} by Eq. (26); 3) Nu'_{ef} by (25); 4) Nu by (7b).

calculations show, the conductive heat transfer is negligible compared to the convective heat transfer, and model (13)-(15) turns, in fact, onto (17)-(20). In conformity with this, at Re > 100 any difference between the coefficients α and α_{ef} disappears.

Within the framework of the approach adopted in the present work, it is easy to clarify how λ_{ef} and A, separately, influence the effective heat transfer coefficient.

Neglect of Conductive Heat Transfer $\{\lambda_{ef} = 0)$. In this case the process is described by Eqs. (10), (11) into which $A\alpha'_{ef}$ must be substituted for the coefficient α_{ef}. The requirement of equivalence gives a formula for calculating α'_{ef}.

$$\alpha'_{ef} = \alpha_{ef}/A\,. \tag{25}$$

The values obtained for the effective heat transfer coefficient α'_{ef} describing the interphase heat transfer in the system where the conductive heat transfer is neglected (the effect of the nonuniformity of the gas flow is not accounted for), are shown in dimensionless form in Fig. 2c.

Neglect of Local Nonuniformmities of the Gas Flow (A = 1). The heat transfer process is described by system (8), (9), where the exchange term has the form $6(1 - \varepsilon)\alpha''_{ef}(T_s - T_f)/d$. The requirement of equivalence of (8), (9) to the new system yields

$$\alpha''_{ef} = A\alpha\,. \tag{26}$$

The values of α''_{ef} are also shown in Fig. 2. They describe the interphase heat transfer process when local nonuniformmities of the gas flow are not taken into consideration. As is seen from Fig. 2, the sensitivity of system (2), (3) to λ_{ef} and A is different. The neglect of λ_{ef} decreases α only insignificantly α – the coefficient α''_{ef}. On the contrary, disregard of the parameter of gas flow nonuniformity A, leads to a substantial (attaining three orders, at small Re) decrease of α, i.e., the coefficient α'_{ef}.

The results obtained make it possible the systematize quantitative regularities of interphase heat transfer in infiltrated granular beds. They allow a proper choice of the governing parameters for calculating nonstationary heat transfer using the two-temperature models.

NOTATION

A, degree of gas flow nonuniformity (at $A = 1$ the flow is completely uniform); C, specific heat; d, particle diameter; K_0, K_1, modified Bessel functions of the second kind of the zeroth and first order; L, height of the granular bed; $\mathrm{Nu} = \alpha d/\lambda_f^k$, Nusselt number; $\mathrm{Pe}_f = C_f \rho_f u L/\lambda_{ef}$, $\mathrm{Pe}_s = C_s \rho_s u_s L/\lambda_{ef}$, $\mathrm{Pe}^* = \alpha^* L^2/\lambda_{ef}$, $\mathrm{Pe} = C_f \rho_f u R^2/\lambda_f^h \varepsilon H$, Peclet numbers; $\mathrm{Pr} = C_f \eta_f/\lambda_f^k$, Prandtl number; $\mathrm{Re} = (u + u_s)d/\nu_f$, Reynolds number; u, u_s, gas and particle velocities based on the empty cross section of the apparatus; t, time; T, temperature; T_0, T^0, inlet temperatures of the gas and the particles; x_i, coordinates; α, interphase heat transfer coefficient; $\alpha^* = 6(1 - \varepsilon)\alpha/d$, $\beta_s = \alpha_{ef}^* L/C_s \rho_s u_s$, $\beta_f = \alpha_{ef}^* L/C_s \rho_s u_s$, $\beta_* = \beta_s/\beta_f$; $\theta = (T - T_0)/(T^0 - T_0)$; $\xi = z/L$; $\eta = r/R$; ε, porosity; κ, thermal conductivity; η_f, dynamic viscosity of the gas; ν_f, kinematic viscosity of the gas; ρ, density. Subscripts and superscripts: s, particles; f, D, gas; ef, effective; ct, contact; w, channel surface; k, molecular; h, pertaining to the gas film.

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